

THE RELATION OF COMBUSTION AND INJECTION
IN A COMPRESSION-IGNITION ENGINE

A THESIS

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By

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THE RELATION OF COMBUSTION AND INJECTION IN A COMPRESSION-IGNITION ENGINE

Summary

Optimum engine performance for minimum weight can be obtained only with controlled combustion. Complete combustion control can be had only by fuel injection during combustion; but in order to utilize fuel injection as a combustion-controlling means, the relationship between combustion and injection must be known. A study of this relationship is made, and it is shown that in a high-performance, compression-ignition engine combustion has characteristics of a chain reaction. A relationship between combustion and injection is developed and its use for engine design is discussed.

Introduction

It is always desirable that internal-combustion engines be as light as is consistent with the function they perform. In large diesel power plants a lighter engine may be less expensive, while in an airplane engine low weight is an absolute necessity.

It can readily be seen that, other things being equal, an engine's weight is proportional to its maximum combustion pressure - the higher the pressure, the more material required to resist the applied forces. Of two equally powerful engines the one with the lower maximum pressure will be the lighter of the two. This principle is well exemplified by the comparative weights of modern automotive gasoline and diesel engines.

It is also apparent that the strength and weight of an engine are not economically utilized unless the engine is operating with a maximum pressure very near to the design-maximum pressure. For example, a modern airplane engine must be stressed to withstand maximum combustion pressures of over 1000 lbs.per sq.in. produced during take-off conditions. This engine under cruising conditions, however, may operate with a maximum pressure of 600 or 700 lbs.per sq.in..

From the foregoing points it may be reasoned that in order to reduce and economically utilize engine weight, two conditions must be fulfilled, as follows:

1. Maximum combustion pressure must be reduced.
2. Maximum combustion pressures must be maintained for ordinary operating, e.g., cruising conditions.

High engine outputs are possible only with high pressures, so the maximum pressure is determined by the maximum power output required.

It can be shown that maximum output for a given limiting maximum pressure is obtained with constant-pressure combustion; e.g., an engine might be supercharged so that the compression pressure is equal to the limiting pressure. On the other hand, engine operation is most efficient when the combustion takes place at constant volume. These two points should be considered along with the maximum allowable pressure in the design of an engine for high performance and minimum weight.

The airplane engine affords a good example for the application of these principles. For an airplane engine the take-off requires a high output, efficiency being a secondary consideration. For cruising operation, efficiency is the chief consideration, and only half to three-quarters of the

maximum output is used.

If an airplane engine were designed for maximum performance and minimum weight in accordance with the foregoing principles, it would develop indicator diagrams similar to those of Fig. 1. Diagram (a) represents take-off operation, while (b) represents cruising. The rate of pressure rise during combustion in diagram (b) is intended to be the maximum consistent with smooth operation.

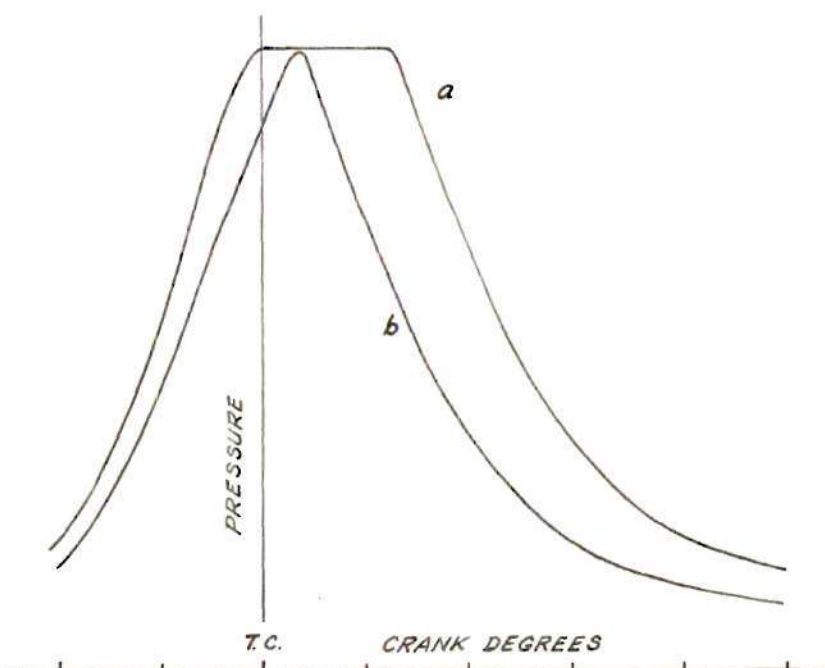


Figure 1

These indicator diagrams are ideal and are possible only with controlled combustion; the combustion must be regulated so that the actual diagrams will have exactly their intended shapes. In short, engines with optimum performance and minimum weight can be had only by controlled combustion.

But how may combustion control be achieved?

In carburetor, spark-ignition, engines the combustion proceeds as the

flame front advances through the mixture. Some slight control of combustion is to be had by special combustion-chamber design; but it is obvious that with all the fuel in the chamber at the moment of ignition, completely controlled combustion is difficult to obtain. The condition is nearly the same when fuel is injected into the manifold during suction or into the cylinder during the first part of the compression stroke. The best possibility for combustion control is with fuel injection during combustion. It is obvious that there can be no combustion if no fuel is present in the chamber and that combustion can proceed no more rapidly than fuel becomes available.

In modern compression-ignition engines a large part of the fuel charge is injected after ignition, so this type of engine has very good possibilities for combustion control by injection. No results of tests with fuel injection during spark-ignited combustion have been found, but the idea has been advanced, and this system, too, would appear to offer good potentialities for combustion control by injection.

The steps in designing a fuel injection system to provide combustion control would be, briefly, as follows:

1. Determination of shape of indicator diagram to give best performance for limiting pressure.
2. Calculation of rate of combustion necessary to produce desired diagram.
3. Determination of rate of injection that will give the required rate of combustion.
4. Design of injection system which will develop the determined rate of injection in the combustion chamber.

In this procedure for injection system design the first two steps can be accomplished by known thermodynamic methods. Step 4 entails the use of data pertinent to the type of injection system considered, but can be carried out by employing known methods and data. For Step 3, however, there is little or no information available. So it is evident that if fuel injection is to be employed as a means of combustion control, the relationship between combustion and injection must be known.

Some general relationships between combustion and injection have been advanced, (References 1 and 2), and the gist of them is that fuel injected after ignition burns with negligible ignition lag. Schweitzer, however, has shown with pilot-injection tests (Reference 3) that the ignition lag of fuel injected into combustion flames is not materially reduced. These references indicate that not only is there no specific relationship available for design use, but there is no complete agreement on even the broadest phases of the relationship between combustion and injection.

This thesis is undertaken for the purpose of developing a definite relationship between combustion and injection in a compression-ignition engine. Characteristics of combustion and injection are taken from actual engine operation and are used as bases for developing the relationship.

Discussion

Engine Test Data

The data used in this study are from National Advisory Committee for Aeronautics tests. The indicator diagrams are those given in Figures 10a, 10b, and 10c of N.A.C.A. Technical Notes No. 569 entitled "Boosted Performance of a Compression-Ignition Engine with a Displacer Piston," Reference 7.

These tests were performed on the N.A.C.A. universal test engine operated as a compression-ignition engine with a displacer piston and a compression ratio of 15.2. The operating speed was 2000 r.p.m. and the inlet pressure was boosted to 7.5 in Hg. above atmospheric pressure.

This test engine is a high-performance, open-chamber engine with good air flow and fuel-air mixing. The progress of combustion with this engine under these conditions might be apparently different from that of other types of engines, e.g., one with a quiescent chamber. This engine, under these test conditions, operates with less ignition lag, and, consequently, less initial acceleration of combustion than might be the case with an unboosted, quiescent-chamber type engine. It is thought, however, that the actual chemical kinetics of the reaction will be of the same type for this and all other open-chamber engines. Therefore, it is believed that developments based on these indicator diagrams should be generally applicable at least to open-chamber, compression-ignition engines under any operating conditions.

Engines of the pre-combustion-chamber and air-cell types may have combustion characteristics which are different from those of open-chamber

engines because the fuel is injected into a restricted volume of air and the flow of the fuel-air mixture during combustion is throttled. In engines of these types all of the air is not immediately available to the injected fuel, but is regulated by chamber pressure changes and orifice sizes. Developments of this study are probably not directly applicable to engines of this type.

High-performance engines must almost necessarily be of the open-chamber type with good air flow. So, since the object of this study is to develop a relationship between combustion and injection for application primarily to high-performance engines, the indicator diagrams used are thought to be very suitable. These diagrams are also of a general nature in that three conditions of combustion are illustrated, i.e., combustion with decreasing pressure, with constant pressure, and with increasing pressure.

Method of Computing Combustion

In studying the relationship between combustion and injection the end points are the engine indicator diagrams and the rate of injection curve. The rate of combustion is intermediate and cannot be read directly from engine test data; it must be computed from the engine indicator diagram. The method of computation used in this study is that developed by Schweitzer (Reference 4). This method gives an expression for the rate of addition of energy by the following equation;

$$\frac{dQ}{d\Theta} = \frac{\frac{c_p}{c_v} P}{\frac{c_p}{c_v} - 1} \frac{dv}{d\Theta} + \frac{v}{\frac{c_p}{c_v} - 1} \frac{dp}{d\Theta}$$

where,

$\frac{dQ}{d\theta}$ = rate of addition of energy per crank degree.

c_p = specific heat at constant pressure.

c_v = specific heat at constant volume.

p = instantaneous chamber pressure.

v = instantaneous chamber volume.

$\frac{dp}{d\theta}$ = rate of pressure rise per crank degree.

$\frac{dv}{d\theta}$ = rate of chamber volume change per crank degree.

The terms of this equation are either measured from the indicator diagram or supplied by specific heat assumptions.

This method was used in Reference 5 and the same considerations of accuracy discussed in that report should apply to the study except that later, more accurate, specific heat data (Reference 6) are employed in this case. A generalization as to the precision may be given by a quotation from Reference 5:

"The variation of the indicated mean effective pressure obtained from the diagrams from the indicated mean effective pressure obtained by adding the brake mean effective pressure to the friction mean effective pressure was, in general, not more than 5 per cent."

Two distinct interpretations of one indicator diagram were made at different times and these results also differed by 5 per cent.

Since this study is concerned more with trends and relative values than with absolute values, this method of combustion computation is believed to be sufficiently exact.

Injection Data

The injection data for these tests were obtained from the N.A.C.A. upon request. It is in the correlation of the injection data to the indicator diagrams that there exists the greatest possibility for error. The rate of injection was, of course, not measured during the operation of the engine; it was obtained with the nozzle mounted in a special measuring apparatus, the injection being made against atmospheric pressure. There is a possibility that against the high pressures in the engine combustion chamber, the injection characteristics were not the same as against atmospheric pressure.

There are factors, however, which indicate that this error is inappreciable, or in any case small. The high valve opening pressure (3500 lbs. per sq.in.) guarantees a nozzle pressure well above the chamber pressure. Since the rate of flow through the nozzle orifices varies approximately as the square root of the pressure difference, the rates against atmospheric and chamber pressures may be roughly compared. The nozzle pressure is taken as equal to the valve opening pressure although it will actually be considered higher during most of the injection.

$$\text{Atmospheric back pressure: } \sqrt{3500 - 15} = 59$$

$$\text{Chamber pressure} = 800 : \sqrt{3500 - 800} = 52 \text{ (Diagrams a and b)}$$

$$\text{Chamber pressure} = 1150 : \sqrt{3500 - 1150} = 48.5 \text{ (Diagram c)}$$

On this score it may be seen that the rate of injection might vary about 20 per cent between the two extremes.

If the difference in back pressure did affect the rate of flow from the nozzle, a given injection quantity would require a greater injection period against a higher back pressure. The engine test data give the

injection period as 28° . The injection period for the same fuel quantity is somewhat indefinite as read from the injection data because of 'dribbling', but the injection data (atmospheric pressure) show a rate of injection at 28° duration of about 13 per cent of the maximum value. This is an effect just opposite that to be expected if the high chamber pressure reduced the rate of injection.

Thus, there is no strictly definite correlation between the measured injection characteristics and the injection characteristics during engine operation. The two sets of characteristics should, however, be very similar so that trends and relative values can be observed with sufficient accuracy.

Discussion of Indicator Line Diagrams and Combustion Calculations

This study is started with the drawing of indicator diagram lines to represent the points recorded by the N.A.C.A. modified-Farnboro indicator. Lines drawn through loci of indicated points at different crank positions give irregular curves (See Fig. 2). These line indicator diagrams, including the irregularities, are used for the calculation of the rate of combustion.

As a general thing the line diagrams would probably be smoothed out, and the weakness of using the irregular diagrams for the analysis is appreciated. The operation of the indicator (Reference 8) in making the points is understood, and it is appreciated that no two cycles have exactly the same combustion characteristics. However, no appreciable errors will be introduced by the irregularities, and it is possible that these irregularities may disclose some trend that is not on the surface apparent or that would be obscured by smoothing out the diagrams.

From these diagrams the combustion calculations are made employing the previously given equation of Schweitzer's method.

$$\frac{dQ}{d\theta} = \frac{\frac{c_p}{c_v} p}{\frac{c_p}{c_v} - 1} \frac{dv}{d\theta} + \frac{v}{\frac{c_p}{c_v} - 1} \frac{dp}{d\theta}$$

The values of c_p/c_v are obtained by the method outlined in Reference 5, but using the specific heat data of Reference 6. The quantities v and $dv/d\theta$ are calculated from actual engine specifications, bore, stroke, compression ratio and connecting-rod length. The rate of pressure rise, $(dp/d\theta)$, and the pressure, p , are measured on the indicator diagrams.

With the units employed, the rate of addition of heat, $dQ/d\theta$, is expressed as inch pounds per crank degree. Since all injection data are in terms of pounds of fuel, $dQ/d\theta$ is translated into $dw/d\theta$ lb. fuel per crank degree. Using the lower heating value of the fuel as 18300 B.t.u./lb., the translating relation is:

$$\frac{dw}{d\theta} = \frac{1}{18300 \times 778 \times 12} \times \frac{dQ}{d\theta}, \quad \text{or}$$

$$\frac{dw}{d\theta} = 0.585 \times 10^{-8} \cdot \frac{dQ}{d\theta}.$$

The total fuel burned up to any crank position is obtained by numerical integration of the rates of combustion. Calculation tabulations and other combustion and injection data are given in Table I. Figure 3 gives graphical representation of the combustion and rate of injection for the respective diagrams.

Since the actual chemical mechanism by which the fuel reacts is unknown, the rate of energy addition, $dQ/d\theta$, cannot be rigorously expressed in terms of the weight-rate of fuel reacting, $dw/d\theta$, by the equation employed. This method of conversion, however, does give a useable, "effective" rate of fuel burning, and the expression is used with this in mind.

Study of Relations between Combustion and Injection

The curves for rate of combustion and rate of injection are somewhat similar in shape. So, since References 1 and 2 suggest that there is a direct relationship between combustion and injection, some simple relationships are investigated.

Figure 4 gives the ratio of (rate of combustion)/(rate of injection) for the diagrams. These curves indicate that no generalization on this basis can be made to describe the relationship between combustion and injection. These figures show definitely, however, that combustion does not immediately occur for the fuel injected after ignition, for which case the ratio would be equal to 1.

The function, (rate of injection)-(rate of combustion) is shown in Fig. 5. These curves are interesting in that each curve shows a tendency to level out at a value of about 0.135×10^{-4} lb./deg. Empirical relations might be developed for these three cases, but it is believed that they would be too specific and not sufficiently applicable in general. Also, there are so many variables that it would be virtually impossible to incorporate them in an empirical expression that would be generally applicable.

The investigation of these and other simple relations indicate that no such relation can be developed for application to the general case of

combustion and injection. With simple empirical relationships not suitable, attention is turned to a rational approach.

Combustion Assumed Bimolecular Collisions

One general approach is to consider the combustion as being the result of collision and reaction of the molecules of fuel and oxygen. This treatment is not proposed as a rigorous analysis of the kinetics of combustion; it is used with the idea that it may be a means, or serve as a foundation of a means, for arriving at a relationship between combustion and injection.

Assume that combustion occurs upon collision of sufficiently activated molecules of fuel and oxygen. The number of collisions of molecules of two unlike gases in a mixture (Reference 9) may be expressed as:

$$Z = N_1 N_2 \sigma_{12}^2 \left[8\pi RT \left(\frac{1}{m_1} + \frac{1}{m_2} \right) \right]^{1/2}, \text{ where:}$$

N = number of molecules per unit volume per second.

σ_{12} = mean molecule diameter,

m = molecule weight.

In order to take account of the fact that all molecules are not sufficiently activated to combine upon collision, the factor, $e^{-\frac{E}{RT}}$, is introduced. This, when multiplied by the total number of collisions gives the number of collisions of activated molecules - an expression of the rate of combustion. Thus,

$$w = N_1 N_2 \sigma_{12}^2 \left[8\pi RT \left(\frac{1}{m_1} + \frac{1}{m_2} \right) \right]^{1/2} e^{-\frac{E}{RT}}$$

where w = rate of combustion.

An absolute expression for this rate of combustion would be difficult to calculate, and because of the variables involved, little accuracy could be hoped for. This equation may, however, serve as indication of how combustion takes place, and to this end the expression is broken down into more recognizable units by the following considerations:

N is proportional to the weight of the substance present,

σ_{12} is constant for all conditions for given substances,

R is constant,

m is constant for each substance,

E is a constant for a given fuel.

A modified equation may be written:

$$R_c = K \frac{w_f w_o}{V^2} \sqrt{T} \cdot e^{-\frac{A}{T}}$$

This is the instantaneous

rate of combustion where:

w_f = weight of unburned fuel present,

w_o = weight of unburned oxygen present,

V = instantaneous chamber volume,

T = absolute temperature $^{\circ}R$,

A = constant, to be determined,

K = constant, to be determined.

When the combustion data obtained from the indicator-card analysis is supplied in this equation, the only two unknowns are the constants, A and

K. The equation may be written with these constants on one side, as:

$$\frac{e}{K} \sqrt{T} = \frac{w_f w_o}{R_c V^2} \sqrt{T}$$

The function of the right side of this equation, for diagram b, is shown in Fig. 6.

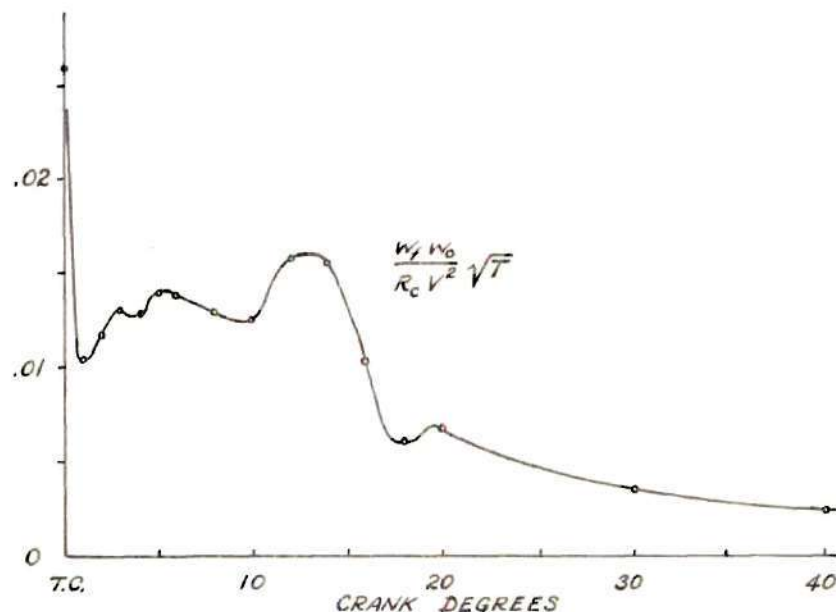


Figure 6

Upon consideration, it may be seen that as T increases, the left side of the equation will decrease, the function having a curve shaped differently from that of the right side. Hence, this relationship is not valid.

Other similar approaches based on the probability of fuel and oxygen molecular collisions were considered, but no satisfactory relationship between combustion and injection was found.

Combustion Studied Stoichiometrically

Another approach in studying the combustion may be taken with the

stoichiometric point of view. Much work was done by F. W. Stevens upon the combustion of gases in a constant-pressure, soap bubble, "bomb". In his works (Reference 10) Stevens indicates that the rate of an explosion reaction may be expressed as

$$s = k [F]^{n_f} [O]^{n_o}$$

where F = concentration of fuel gas,
 O = concentration of oxygen,
 n = molecules of fuel reacting,
 k = reaction constant.

Stevens also adapts this expression to account for different pressures and the addition of diluent inactive gases (References 11 and 12). Assuming that the engine fuel oil can be expressed by the formula $C_{16}H_{34}$, the chemical reaction may be written as



This equation gives the reaction order as being 51. Stevens' equations incorporating factors for pressure and diluent gas were modified to employ terms derived from the indicator-diagram combustion computations, and the engine combustion was compared with the modified equation.

The agreement was not good, the chief discrepancy being that the effect of the oxygen concentration as employed in the equation was of much greater importance than it actually proved to be for the engine combustion. Also, very large exponents (over 100) entered into the computations and made calculations impractically laborious for any design work. This approach was unproductive in developing a relationship between combustion and

injection.

During the foregoing investigations certain features of combustion were observed to be contrary to general chemical reaction theories. One feature is that the temperature has little effect upon the progress of combustion. Another feature is that the oxygen concentration plays little part in determining the progress of combustion. Conversely, the fuel concentration seems to be quite important.

Development Based on Chain Reaction Theory

Since none of the previously discussed approaches offer recognizable means of relating the combustion and injection, another method of attack is tried. This method is based upon the assumption that combustion is a chain reaction and has characteristics as such. As justification for this assumption, a statement may be quoted from N. Semenov (Reference 13):

"It appears that the kinetics of any chemical reaction is that of a chain mechanism, and that the particular cases involving, for instance, reactions with very short chains or those elementary reactions requiring but little energy of activation (the reaction of some free radicals and atoms), which can be interpreted without recourse to chains are very rare indeed..."

The theory of chain reaction is a comparatively recent development and fundamentals are still being established. An excellent definition is given by Semenov (Reference 13a),

"The theory of chain reactions does not differ essentially from classical kinetics except for the additional assumption that the chemical energy, set free in an elementary act, is not entirely transformed into heat, but is spent, at least partially, in the formation of intermediate products carrying considerable energy. Owing to their extreme capacity for chemical change, these products readily react with the original substances and are renewed again and again at the expense of the released energy. It is this reiterated process

which constitutes the formation of a reaction chain. It appears as if, at the cost of the liberated energy, the reaction cleared its own way, transforming the comparatively stable molecules of the original substances into other modifications more liable to chemical changes, and thus overcoming 'the chemical resistance' of the system."

There are several types and combinations of types of chain reactions and everything is not yet completely understood about them. The type of chain reaction that takes place is determined by the substances reacting and the conditions under which they react. The chains include products intermediate between the initial substances and the products of the completed reaction, and there are relatively few chain reactions for which the complete mechanism can be satisfactorily detailed. On the whole, the details of chain reactions are quite complex.

Semenoff has made a study of a great many reactions and in the summary of his book (Reference 13b) states,

"All the reactions examined above differ essentially in their chemical behavior. Still, these chemically different types of reactions are governed by the same kinetical laws since the kinetical course of any reaction is determined only by a few parameters, irrespective of the mechanism involved."

Semenoff shows that the initial acceleration observed for chain reactions obeys one of two fundamental laws. Under the second of these two laws comes, among other reactions, the oxidation of such hydrocarbons as methane, ethane, ethylene, and benzene. He further establishes a general law which expresses for the whole course of these reactions the dependence upon time of the percentage change, ξ , and the reaction rate $w = \frac{d\xi}{dt}$, namely,

$$\xi = \frac{100}{1 + e^{-\phi\theta}} \quad \text{and} \quad w = \frac{100 \phi e^{-\phi\theta}}{(1 + e^{-\phi\theta})^2} \cdot \quad (\text{Reference 13c})$$

ϕ = a factor for initial conditions,

Θ = time from half life, i.e., when $\xi = 50$ per cent.

A graphical representation of these expressions is given in Fig. 7.

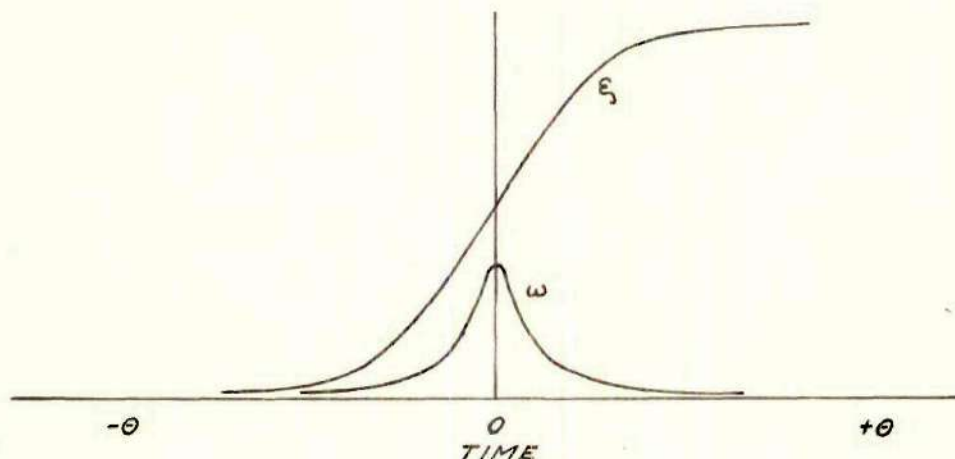


Figure 7

These curves have forms respectively similar to those of the rate of combustion and amount of fuel burned as computed from the engine indicator diagrams. These expressions are taken as a basis for developing a relation between combustion and injection.

As a preliminary step in attempting to use these expressions for the development of a relationship between combustion and injection, it is desirable to ascertain whether or not the combustion in the engine is such that these expressions are applicable. In other words, is the engine combustion process a chain reaction of the type indicated by these expressions.

These expressions, as developed by Semenoff, are based upon experiments in which the mixtures of the reacting substances were complete before the reaction started, e.g., a mixture of gas and oxygen in a combustion bomb. It is the initial condition at the moment of ignition that determines

the important factor, ϕ . The case of combustion in the compression-ignition engine is obviously not analogous, so the expressions cannot be applied outright.

There is, however, a part of the engine combustion cycle where conditions are similar to those for which the expressions were developed. This is the latter part of combustion where practically all of the fuel is in the combustion chamber and the rate of injection has dropped to a negligible value. In the expression

$$\xi = \frac{100}{1 + e^{-\phi\Theta}}$$

everything but ϕ is known from the indicator-diagram combustion calculations. This equation may be expressed as

$$e^{-\phi\Theta} = \frac{100}{\xi} - 1$$

and ϕ may be simply found.

The total amount of fuel burned has been computed. At the crank position where half of the total fuel is burned, $\Theta = 0$; for convenience Θ is taken in terms of crank degrees. With the known values supplied, ϕ can be found. If the engine combustion is expressible by these equations, it is obvious that ϕ should be found a constant at every stage of combustion. Tabulations of the solutions for ϕ are given in Table II.

Everything considered, the respective values of ϕ for each indicator diagram are satisfactorily constant. This means that the combustion in the

engine, at least the part of combustion considered, has the characteristics of a chain reaction and is expressible in terms of the equations developed by Semenov.

It is not to be expected that the values of ϕ for all three indicator diagrams should be the same. Although the meaning of ϕ is not strictly the same as in the experiments considered by Semenov, it still refers to the initial conditions. In the case of the compression-ignition engine, the initial conditions for some of the fuel may be at a point where the combustion is well under way. It is obvious that the conditions of temperature, pressure, and fuel concentration are not the same at corresponding combustion times for the three indicator diagrams.

It is apparent that these values found for ϕ are not applicable in expressing the reaction progress while injection is taking place and this has been checked by actual trial. This is also made apparent by the observation that, in diagram b for example, the first half of the fuel burns in 24° while the second half of the combustion requires 38° . In the expressions of Semenov equal times are given for both halves of the reaction.

What actually takes place is that as long as fuel is being injected new chains are initiated so that the value of ϕ is built up, so to speak. The value of Θ during injection is meaningless since the reaction progress is not a function of time alone. During injection the effective variable is the product, $\phi\Theta$, later designated as Ψ .

This variable, Ψ , is itself in some manner dependent upon time and injection. If this relation between Ψ and the injection can be found, the problem of relating the combustion and injection is solved. An attempt is

therefore made to get an expression for Ψ in terms of the combustion and injection values computed from the indicator diagrams.

The expression

$$\xi = \frac{100}{1 + e^{-\Psi}}$$

may be revised to express the amount reacted,

$$x = \frac{x_{\infty}}{1 + e^{-\Psi}}$$

where x_{∞} = final total amount reacted.

In terms of the values computed from the indicator cards this is expressed

$$w_b = \frac{w_i}{1 + e^{-\Psi}} \quad (1) \quad \text{where } w_b = \text{amount of fuel burned up to any instant,}$$

$$w_i = \text{amount of fuel injected up to the instant.}$$

This leads to

$$e^{-\Psi} = \frac{w_i - w_b}{w_b}, \quad \text{or,}$$

$$e^{-\Psi} = \frac{w_f}{w_b} \quad (2) \quad \text{where } w_f = \text{amount of unburned fuel present.}$$

Taking the log of both sides gives:

$$\Psi = \text{Log } \frac{w_b}{w_f} \quad (3)$$

Differentiating with respect to time (crank degrees) gives:

$$\frac{d\psi}{d\theta} = \frac{1}{w_b} \frac{dw_b}{d\theta} - \frac{1}{w_f} \frac{dw_f}{d\theta}, \quad \text{or,}$$

$$\frac{d\psi}{d\theta} = \frac{1}{w_b} \frac{dw_b}{d\theta} - \frac{1}{w_f} \left[\frac{dw_1}{d\theta} - \frac{dw_b}{d\theta} \right].$$

But, $\frac{dw_b}{d\theta} = R_c$, the rate of combustion, and

$\frac{dw_1}{d\theta} = R_i$, the rate of injection.

So the expression for $\frac{d\psi}{d\theta}$ may be written:

$$\frac{d\psi}{d\theta} = \frac{R_c}{w_b} - \frac{R_i - R_c}{w_f} \quad (4)$$

Integrating to get ψ would give:

$$\psi = \int \left[\frac{R_c}{w_b} - \frac{R_i - R_c}{w_f} \right] d\theta + C \quad (5)$$

or,

$$\psi = \int \left[\frac{w_f R_c + w_b R_c - w_b R_i}{w_f w_b} \right] d\theta + C,$$

and separating terms,

$$\psi = \int \frac{w_i R_c}{w_f w_b} d\theta - \int \frac{R_i}{w_f} d\theta + C. \quad (6)$$

But the expression for ψ is given in Eq.(3). Equating these expressions gives

$$\text{Log } \frac{w_b}{w_f} = \int \frac{w_i R_c}{w_f w_b} d\theta - \int \frac{R_i}{w_f} d\theta + C. \quad (7)$$

C is a constant of integration which is determined by engine conditions.

In this equation there are only two variables, R_c and R_i ; and both of them are expressed in terms of time (or crank degrees). The integrals cannot be solved outright, but can be satisfactorily calculated by numerical integration.

For the engine combustion, all of the values in the equation are known, or have been computed from the indicator diagrams, except the constant of integration, C . Employing numerical integration for the integrals, the validity of this equation can be tested by solving for C . The calculation tabulations and values of C are given in Table III.

For respective indicator cards the values found for C are remarkably consistent. As would be expected, the values of C are different for the individual indicator diagrams because of the different conditions prevailing in each. But the fact that C checks out so consistently close to a constant value for each indicator diagram is strong evidence that the equation developed does express a valid relation between combustion and injection.

To make the equation complete for design use it would be desirable to have an expression for evaluating C . This constant is evidently determined by engine conditions mainly at the moment of ignition, but no relationships are apparent from the cases considered. One chief factor is the ignition lag since it and the initial rate of injection determine the weight of fuel, w_f , present at the moment of ignition. The ignition lag itself, of course, is dependent upon temperature, pressure, fuel characteristics, turbulence, and probably fuel concentration.

Use of Equation for Design

Since Eq. (7) has no exact solution for the integrals, it is admittedly awkward to handle for design purposes. But its use is possible.

For injection system design, as outlined before, the rate of combustion, R_c , and, consequently, the weight of fuel burned, w_b , are initially determined to give the desired shape to the indicator card. The quantity to be found by use of the developed relationship is R_i , the rate of injection which will produce the desired rate of combustion. With given values of R_c and w_b , the values of w_i and w_f are determined when R_i is known.

For use, then, the equation requires a trial-and-error solution for R_i , numerical integration being employed for evaluating the integrals. Assuming that computations are made in steps of 1 crank degree, the expression for R_i at any crank position, Θ , would be:

$$\left[\frac{R_i}{w_f} \right]_{\Theta} = \int_{\Theta}^{\Theta-1} \frac{w_i R_c}{w_f w_b} d\Theta + \left[\frac{w_i R_c}{w_f w_b} \right]_{\Theta} - \int_{\Theta}^{\Theta-1} \frac{R_i}{w_f} d\Theta + \text{Log}_e \left[\frac{w_f}{w_b} \right]_{\Theta} + C \quad (8)$$

or

$$R_i = (w_f)_{\Theta} \left[\int_{\Theta}^{\Theta-1} \frac{w_i R_c}{w_f w_b} d\Theta + \left[\frac{w_i R_c}{w_f w_b} \right]_{\Theta} - \int_{\Theta}^{\Theta-1} \frac{R_i}{w_f} d\Theta + \text{Log}_e \left[\frac{w_f}{w_b} \right]_{\Theta} + C \right] \quad (9)$$

where

R_i = lb. per crank degree,

R_c = lb. per crank degree,

w_i = lb.,

w_b = lb.,

w_f = lb.

If a test can be run on the engine with design conditions prevailing at the moment of ignition, the developed indicator diagram may be analyzed for combustion and the constant C may be found from Eq. (7) and then used for computing R_1 for the final injection system design.

The calculations for R_1 must be terminated with allowances being made for injection cut-off, so that the total weight of fuel injected will be that which is consistent with the intended air-fuel ratio. It will be noted that after injection has ceased, much of the fuel is yet to burn. Its combustion will take place, uncontrolled, according to the laws governing the prevailing chain reaction.

Limitations of Combustion Control by Injection

For maximum power it is desirable that the entire combustion be controlled. This is not completely possible if fuel is injected only up to the amount which will give a fuel-air ratio of, say, 15. From the data available it is not possible to determine whether or not the derived relation for combustion and injection will hold for the latter part of combustion with very rich fuel-air mixtures.

If the relation should hold good, it would be possible to have injection-controlled combustion up to the point where all of the air was utilized. The final combustion products would not, of course, be those of complete combustion, i.e., CO_2 and H_2O . The fuel consumption would be high, but would be an acceptable price to pay for momentary high output, as required, for instance, by an airplane engine during take-off.

Conclusions

1. Combustion in the high-performance, compression-ignition engine considered in this study has the characteristics of a chain reaction. When injection has ceased, the combustion is expressible by the equation developed by Semenov in which time is the only variable.

2. During injection the combustion reaction is a function of both time and the rate of injection. Based upon the laws propounded by Semenov, a relation between combustion and injection is developed. This relation is:

$$\text{Log } \frac{w_b}{w_f} = \int \frac{w_1 R_c}{w_f w_b} d\theta - \int \frac{R_1}{w_f} d\theta + C.$$

This relation is thought to be valid for the combustion in all open-chamber, compression-ignition engines. One remarkable feature of the expression is that all differences in fuels and engine operating conditions are 'accounted for' in the constant, C.

3. The developed relation can be modified for use in computing the rate of injection for actual engine design. By the use of this combustion-injection relationship in designing fuel injection systems, engines of optimum performance and minimum weight should be made possible.

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Table Ia
Combustion Calculations for Indicator Diagram a.

Crank Deg.	$\frac{C_p}{C_v}$	$\frac{dv}{ds}$ in ³ /deg.	A	V	$\frac{dp}{ds}$ cu. in.	B	$\frac{dq}{ds}$ in. lb./deg.	$\frac{dw}{ds} = R_c$ in. lb./deg.	w_b lb.	w_l lb.	w_f lb.
5	1.343	0.1352	404	10.02	-12	-351	53	.0031x10 ⁻⁴	.00016x10 ⁻⁴	0.35x10 ⁻⁴	0.35x10 ⁻⁴
6	1.343	0.162	477	10.17	-10	-296	181	.0106	.00696	0.483	0.490
8	1.337	.215	629	10.55	-4.5	-141	488	.0285	.0461	0.784	0.830
10	1.332	.269	796	11.09	+2.0	+66.7	862.7	.0505	.1251	1.053	1.178
12	1.326	.321	970	11.67	+2.2	+78.8	1048.8	.0614	.2370	1.327	1.564
14	1.321	.372	1139	12.29	0	0	1139.	.0666	.3650	1.616	1.981
16	1.315	.424	1298	13.22	-16	-671	627.	.0367	.4683	1.982	2.450
18	1.310	.475	1436	14.05	+1.2	+54.4	1490.4	.0871	.5921	2.350	2.942
20	1.304	.525	1624	15.08	+4.6	+228.	1852.	.1085	.7877	2.61	3.398
22	1.298	.570	1810	16.20	+3.0	+163.	1973.	.1154	1.0107	2.82	3.831
24	1.293	.616	1990	17.40	-2.0	-118.7	1871.3	.1097	1.2359	2.84	4.076
26	1.288	.662	2100	18.74	-24.	-156.3	1944.7	.114	1.4596	2.78	4.240
28	1.282	.701	2162	20.20	-7.7	-551	1611.	.0944	1.6680	2.66	4.328
30	1.277	.753	2312	21.76	-4.6	-378.	1934.	.1132	1.8756	2.53	4.405
32	1.272	.794	2430	23.32	-9.0	-772.	1658.	.0970	2.0858	2.35	4.436
34	1.266	.834	2480	25.00	-15.0	-1410.	1070.	.0626	2.2454	2.2	4.445
36	1.261	.872	2485	26.7	-15.0	-1535.	950.	.0556	2.3636	2.10	4.46
40	1.25	.948	2485	30.51	-14.5	-1770.	715.	.0418	2.5584	1.90	4.46
45	1.25	1.030	2370	35.41	-13.0	-1840.	530.	.0310	2.7404	1.72	4.46
50	1.25	1.070	2140	40.63	-12.0	-1950.	190.	.0111	2.8457	1.615	4.46
55	1.251	1.147	1990	47.43	-10	-1890.	100.	.00585	2.8881	1.57	4.46
60	1.252	1.190	1772	52.31	-7.5	-1562.	210.	.0123	2.9366	1.53	4.46
65	1.253	1.220	1584	58.38	-7.0	-1613.	29.	.0017	2.9631	1.50	4.46

$$A = \frac{C_p \cdot P}{C_v} \cdot \frac{dv}{ds}$$

$$B = \frac{V}{C_p - 1} \cdot \frac{dp}{ds}$$

Table Ib
Combustion Calculations for Indicator Diagram b.

Crank Deg.	$\frac{c_p}{c_v}$	$\frac{dv}{d\theta}$	A	V	$\frac{dp}{d\theta}$	B	$\frac{dQ}{d\theta}$	$\frac{dw}{d\theta} = R_c$	w_b	w_i	w_f
		in ³ /deg.		cu. in.			in. lb./ deg.	in. lb./ deg.	lb.	lb.	lb.
0	1.343	0	0	9.68	0.7	19.8	19.8	0.00116x _A 10 ⁻⁴	0.00058x _A 10 ⁻⁴	0.04x10 ⁻⁴	.0395x10 ⁻⁴
1	1.340	0.02715	84.9	9.694	0.7	20.0	104.9	0.00614	0.00423	0.1 "	.0958
2	1.338	0.0542	170.5	9.736	0.7	20.2	190.7	0.01116	0.01288	0.000021	.197
3	1.335	.0813	257.	9.803	0.7	20.5	277.5	.01624	.02658	.000036	.333
4	1.333	.1083	345.	9.898	0.7	20.8	365.8	.0214	.04540	.0000495	.45
5	1.330	.1352	434.	10.02	0.7	21.3	455.3	.0266	.06940	.000066	.590
6	1.328	.162	523.	10.17	0.7	21.7	544.7	.0319	.09865	.000083	.731
8	1.323	.215	704.	10.555	0.7	22.9	726.9	.0425	.17305	.000118	1.007
10	1.318	.269	891.	11.09	0	0	891.0	.0521	.26765	.000156	1.292
12	1.313	.321	1064.	11.67	- 7.14	- 266.	798.	.0467	.36645	.000198	1.614
14	1.308	.372	1221.	12.29	- 8.13	- 324.	897.	.0525	.46565	.000245	1.98
16	1.303	.424	1386.	13.22	0	0	1386.	.0811	.59925	.000295	2.35
18	1.298	.475	1605.	14.05	+ 14.3	674.	2279.	.1334	.81375	.000342	2.61
20	1.293	.525	1858.	15.08	0	0	1858.	.1087	1.05585	.000383	2.774
25	1.280	.641	2210.	18.03	- 12.5	- 805.	1405.	.0822	1.53285	.000430	2.77
30	1.268	.753	2500.	21.76	- 14.3	-1161.	1339.	.0784	1.93435	.000443	2.50
35	1.256	.855	2596	25.79	- 16.0	-1612.	984.	.0576	2.27435	.000444	2.17
40	1.243	.948	2590.	30.51	- 16.0	-2009.	581.	.0340	2.50335	.000445	1.95
45	1.244	1.030	2440.	35.41	- 14.0	-2030.	410.	.0240	2.64835	.000445	1.80
50	1.244	1.070	2170.	40.63	- 11.0	-1833.	337.	.0197	2.75785	.000445	1.69
55	1.245	1.147	2040.	47.43	- 9.7	-1876.	164.	.00960	2.831	.000445	1.62
60	1.245	1.190	1820.	52.31	- 8.3	-1772.	48.	.00281	2.862	.000445	1.59
65	1.246	1.220	1620	58.38	- 7.0	-1660	- 40.	.00234	2.865	.000445	1.59

Table Ic
Combustion Calculations for Indicator Diagram c.

Crank Deg.	$\frac{p}{p_c}$	$\frac{dv}{ds}$ in. ³ /deg.	A	V	$\frac{dp}{ds}$	B	$\frac{dq}{ds}$ in.lb./deg.	$\frac{dw}{ds} = R_c$ in.lb./deg.	w_b lb.	w_1 lb.	w_f lb.
-5	1.343	-0.1352	400	10.02	+ 15	+ 437	37	0.00215x ⁻⁴ ₁₀	0.00107x ⁻⁴ ₁₀	0.35x10 ⁻⁴	0.349x10 ⁻⁴
-4	1.338	-0.1083	330	9.898	18	526	196	.0115	.00789	.49 "	.482
-3	1.334	-0.0813	257	9.803	28	821	564	.033	.0302	.000066	.63
-2	1.330	-0.0542	181	9.736	31.2	920	739	.0432	.0683	.000083	.762
-1	1.325	-0.02715	95.7	9.694	31.2	930	834	.0488	.1143	.000099	.875
0	1.321	0	0	9.68	31.2	941	941	.0550	.1662	.000117	1.005
+1	1.316	0.02715	104.6	9.694	31.2	957	1062	.0621	.2247	.000136	1.135
2	1.312	.0542	217	9.736	31.2	974	1191	.0697	.2906	.000156	1.27
3	1.307	.0813	342	9.803	31.2	995	1337	.0782	.3645	.000176	1.397
4	1.303	.1083	473	9.898	25.6	836	1309	.0766	.4419	.000198	1.535
5	1.298	.1352	613	10.02	24.4	820	1433	.0838	.5221	.000223	1.71
6	1.294	.162	759	10.17	23.8	823	1582	.0926	.6103	.000245	1.845
8	1.285	.215	1077	10.555	23.8	882	1959	.1146	.8175	2.94x10 ⁻⁴	2.13
10	1.276	.269	1424	11.09	6.0	241	1665	.0975	1.0296	3.42	2.40
12	1.267	.321	1752	11.67	0	0	1752	.1025	1.2296	3.90	2.67
14	1.258	.372	2080	12.29	4	190	1890	.1106	1.4427	4.21	2.77
16	1.249	.424	2400	13.22	- 12.5	- 663	1736	.1015	1.6548	4.39	2.74
18	1.240	.475	2670	14.05	- 20.	- 1171	1499	.0877	1.8440	4.50	2.65
20	1.240	.525	2840	15.08	- 21.5	- 1352	1488	.0871	2.0188	4.56	2.54
25	1.241	.641	3010	18.03	- 30.	- 2245	765	.04475	2.3488	4.61	2.26
30	1.241	.753	2050	21.76	- 24.	- 2170	880	.0515	2.5893	4.64	2.05
35	1.242	.855	2940	25.79	- 23.5	- 2504	436	.0255	2.7818	4.66	1.87
40	1.242	.948	2685	30.51	- 19.	- 2393	292	.0171	2.8883	4.66	1.77
45	1.243	1.030	2480	35.41	- 15.1	- 2200	280	.0164	2.9720	4.66	1.69
50	1.243	1.070	2190	40.63	- 12.	- 2100	90	.0053	3.0262	4.66	1.64
55	1.244	1.147	1990	47.43	- 10.2	- 1980	10	.00058	3.0409	4.66	1.62

Table II

Solutions for ϕ of equation $\xi = \frac{100}{1 + e^{-\phi\theta}}$

	Crank Degrees	$\frac{\xi}{100}$	$e^{\phi\theta}$	$\phi\theta$	θ	ϕ
Diagram a. —— Half fuel burned, or $\theta = 0$, at 26° crank.	34	0.76	3.13	1.14	8	0.144
	36	.80	3.95	1.37	10	.139
	40	.86	6.33	1.85	14	.133
	45	.93	12.34	2.51	19	.133
	50	.96	24.4	3.20	24	.134
	55	.98	38.5	3.65	29	.127
	60	.99	100.	4.6	34	.136
Diagram b. —— Half fuel burned, or $\theta = 0$, at 24° crank.	30	0.68	2.09	.73	6	0.122
	35	.79	3.85	1.35	11	.122
	40	.88	7.00	1.94	16	.121
	45	.92	12.04	2.49	21	.119
	50	.96	25.6	3.25	26	.125
Diagram c. —— Half fuel burned, or $\theta = 0$, at $14\frac{1}{2}^\circ$ crank.	25	0.77	3.40	1.22	10.5	0.117
	30	.85	5.73	1.74	15.5	.112
	35	.92	10.75	2.47	20.5	.121
	40	.95	18.9	2.94	25.5	.116
	45	.98	41.7	3.73	30.5	.122

Diagram	Average	Per cent Maximum Deviation from Average	
a	0.135	+ 6.7	- 6.0
b	0.122	+ 2.5	- 2.5
c	0.118	+ 3.5	- 1.7

Table IIIa

Solution for C of Equation:

$$\text{Log } \frac{w_b}{w_f} = \int \frac{w_1 R}{w_f w_b} d\theta - \int \frac{R_1}{w_f} d\theta + C$$

Crank Deg.	$\frac{w_1 R_c}{w_f w_b}$	$\frac{R_1}{w_f}$	$\int \left[\frac{w_1 R_c}{w_f w_b} - \frac{R_1}{w_f} \right] d\theta$	$\text{Log } \frac{w_b}{w_f}$	C
5	1.940	0.377	1.398	4.24	- 5.638
6	1.545	.313	3.077	2.835	- 5.912
8	0.654	.207	3.797	2.13	- 5.927
10	.452	.179	4.228	1.725	- 5.953
12	.306	.148	4.484	1.488	- 5.972
14	.224	.126	4.555	1.445	- 6.000
16	.097	.124	4.601	1.378	- 5.979
18	.184	.111	4.763	1.196	- 5.959
20	.179	.0904	4.946	1.028	- 5.974
22	.155	.0613	5.127	0.833	- 5.960
24	.126	.0391	5.305	0.644	- 5.949
26	.119	.0284	5.472	0.466	- 5.938
28	.092	.0164	5.652	0.300	- 5.952
30	.115	.01105	5.967	0.0198	- 5.947
34	.056	.0025	6.236	.298	- 5.938
40	.0383	.0021	6.394	.466	- 5.928
45	.029	.002	6.484	.566	- 5.918
50	.0108	.002	6.515	.610	- 5.905
55	.0057	.002			

Table IIb

Solution for C of Equation:

$$\text{Log } \frac{w_b}{w_f} = \int \frac{w_i R_c}{w_f w_b} d\theta - \int \frac{R_i}{w_f} d\theta + C$$

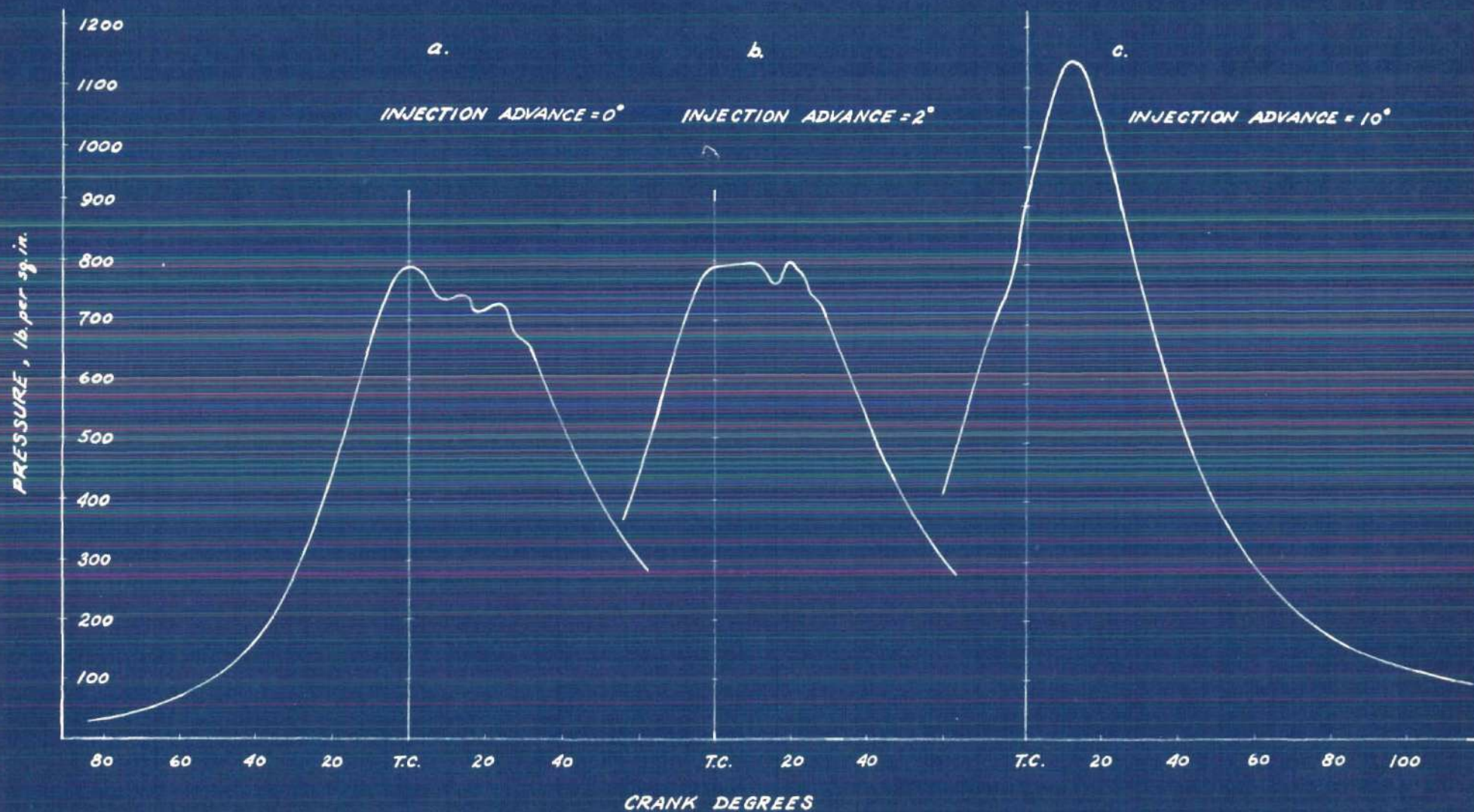
Crank Deg.	$\frac{w_i R_c}{w_f w_b}$	$\frac{R_i}{w_f}$	$\int \left[\frac{w_i R_c}{w_f w_b} - \frac{R_i}{w_f} \right] d\theta$	$\text{Log } \frac{w_b}{w_f}$	C
0	2.00	0.456			
2	0.923	.563	1.904	- 2.73	- 4.634
4	.518	.335	2.447	- 2.29	- 4.737
6	.367	.222	2.775	- 2.00	- 4.775
8	.288	.188	3.020	- 1.76	- 4.780
10	.234	.152	3.202	- 1.574	- 4.776
12	.156	.126	3.314	- 1.482	- 4.796
14	.139	.124	3.359	- 1.45	- 4.809
16	.170	.111	3.433	- 1.366	- 4.799
18	.215	.091	3.616	- 1.165	- 4.781
20	.142	.062	3.820	- 0.965	- 4.785
25	.0832	.021	4.176	- 0.590	- 4.766
30	.0718	.0032	4.503	- 0.256	- 4.759
35	.0520	.0019	4.802	+ 0.047	- 4.755
40	.0310	.0020	5.000	0.249	- 4.751
45	.0224	.0022	5.123	.387	- 4.746
50	.0188	.0023	5.215	.490	- 4.725
55	.0092	0	5.279	.559	- 4.720

Table IIIc

Solution for C of Equation:

$$\log \frac{w_b}{w_f} = \int \frac{w_1 R_c}{w_f w_b} d\theta - \int \frac{R_1}{w_f} d\theta + C$$

Crank Deg.	$\frac{w_1 R_c}{w_f w_b}$	$\frac{R_1}{w_f}$	$\int \left[\frac{w_1 R_c}{w_f w_b} - \frac{R_1}{w_f} \right] d\theta$	$\log \frac{w_b}{w_f}$	C
-5	2.020	0.321			
-4	1.482	.313	1.434	-4.111	-5.545
-2	0.689	.213	3.079	-2.413	-5.492
0	.369	.188	3.736	-1.803	-5.539
2	.295	.154	4.158	-1.473	-5.531
4	.223	.133	4.289	-1.246	-5.535
6	.205	.133	4.451	-1.107	-5.558
8	.193	.122	4.594	-0.958	-5.552
10	.135	.1025	4.698	.846	-5.544
12	.122	.086	4.766	.775	-5.541
14	.116	.057	4.861	.660	-5.521
16	.101	.029	4.992	.506	-5.498
18	.0808	.0162	5.129	.364	-5.493
20	.0775	.0110	5.260	.229	-5.489
25	.0388	.0027	5.517	.0392	-5.478
30	.0450	.0020	5.715	.233	-5.482
35	.0228	.0021	5.875	.396	-5.479
40	.0155	.0023	5.960	.490	-5.470
45	.0152	.0018	6.026	.564	-5.462



INDICATOR DIAGRAMS : 2000 r.p.m. ; 7.5 in. Hg. Boost ; Compression Ratio = 15.2

Figure 2.

RATES OF COMBUSTION & INJECTION, 16×10^{-5} per deg : AMOUNT BURNED, 16×10^{-4}

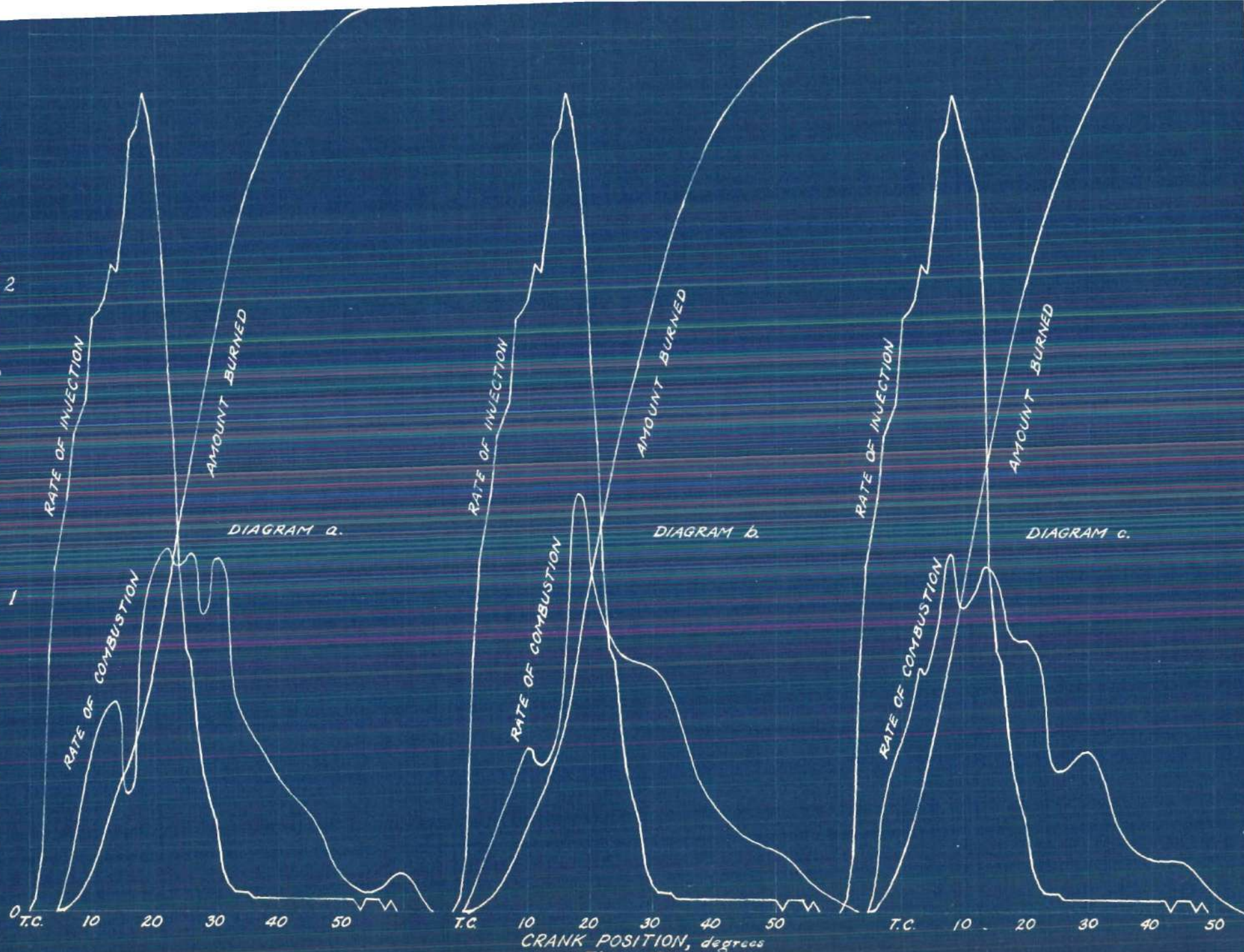


Figure 3

RATIO of $\frac{\text{RATE OF COMBUSTION}}{\text{RATE OF INJECTION}}$

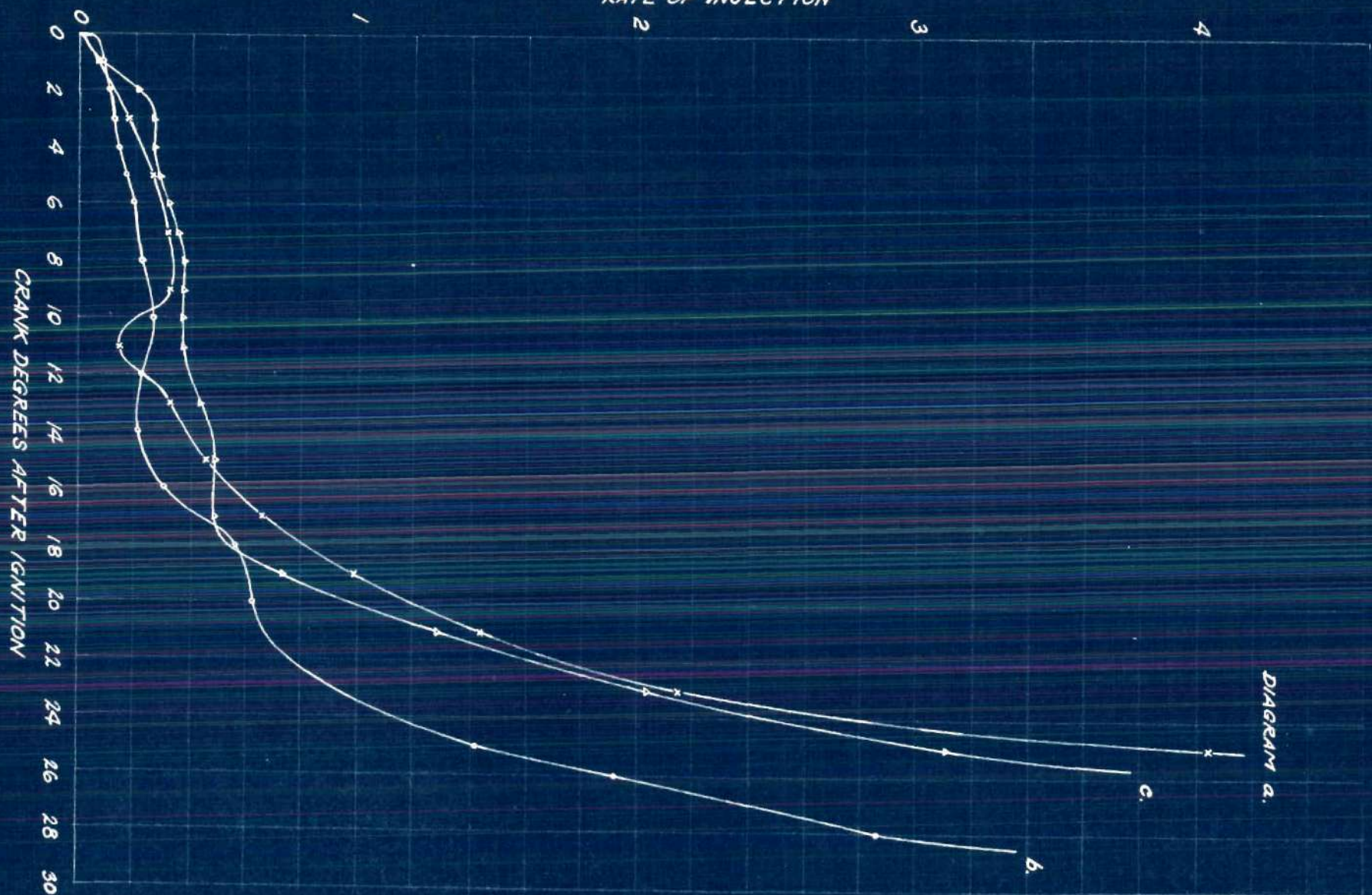


DIAGRAM a.

Figure 4

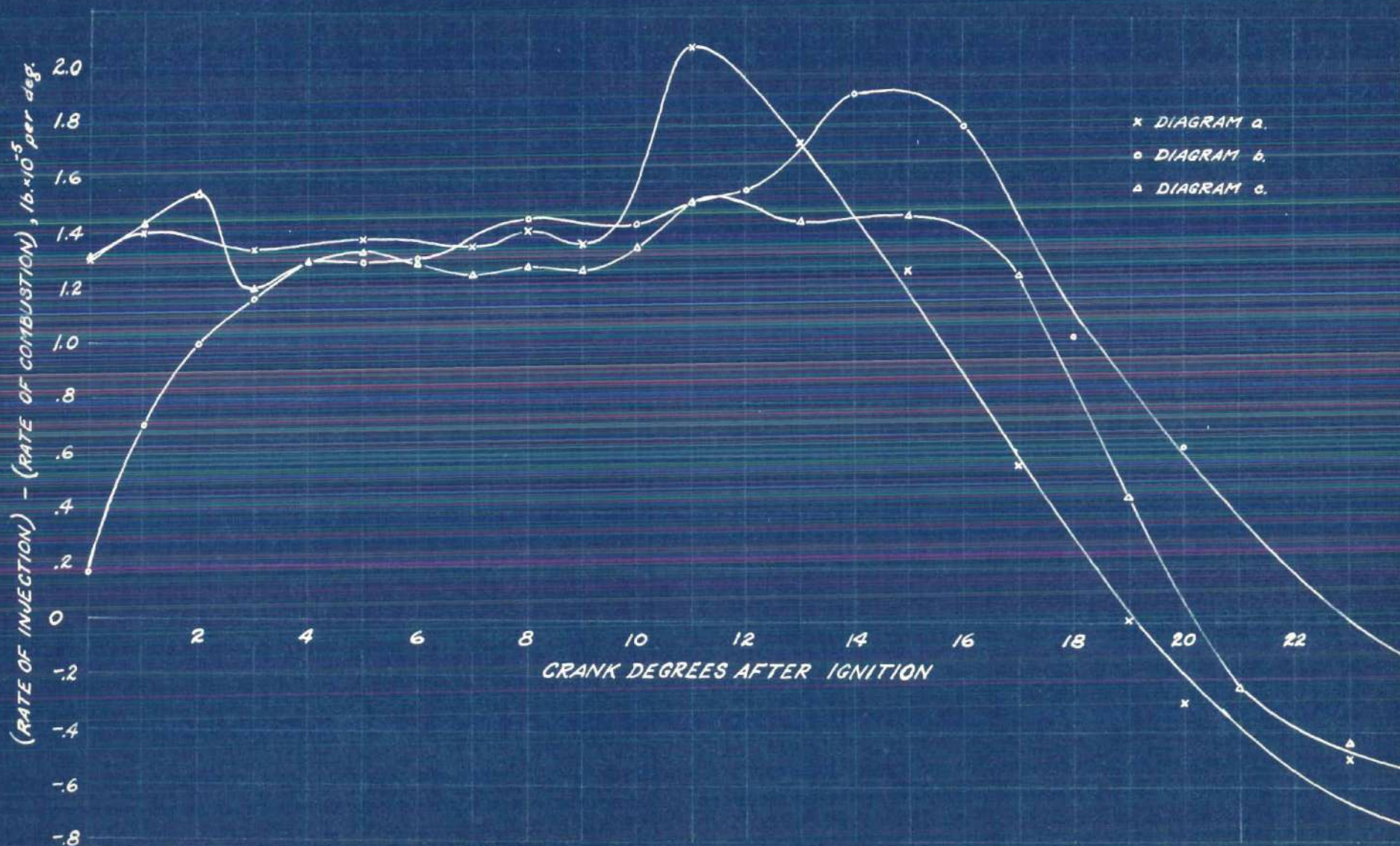


Figure 5